

## An Improved Dielectric Resonator Method for Surface Impedance Measurement of High-Tc Superconductors.

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**Abstract**—An improved dielectric resonator method is proposed to measure temperature characteristics of surface impedance  $Z_s$  of superconductors automatically with high resolution and with good reproducibility. Perturbation formula for this resonator is derived to determine  $Z_s$  from measured values of resonant frequency and unloaded Q. Some measured results verify the usefulness of this method. A consideration of the effect of surface roughness enables one to compare measured temperature dependence of the complex conductivity with the BCS and three-fluid models.

### I. INTRODUCTION

To design microwave components using high-Tc superconductors, the surface impedance  $Z_s = R_s + jX_s$ , where  $R_s$  is the surface resistance and  $X_s$  is the surface reactance, is one of the most important parameters. A TE<sub>011</sub> mode cavity method[1] and a TE<sub>011</sub> mode dielectric rod resonator method [2] are known to measure  $Z_s$  in microwave region. However there are two experimental difficulties in these methods; one is the poor reproducibility of measured  $Z_s$  values and the other is the limit of  $R_s$  measurement below  $10^{-3} \Omega$  at 10 GHz. The former occurs from uncertain mechanical contacts and the latter occurs from strong influence of the conductor plate loss on unloaded Q.

To overcome these difficulties, this paper proposes an improved method using a dielectric-loaded cavity. A perturbation formula for this resonator is derived. The accuracy of  $Z_s$  measurement by this method is compared with those by two methods described above. For some high-Tc superconductors the  $Z_s$  values are measured by this method at 10GHz. The effect of the surface roughness on  $Z_s$  measurement is discussed. Measured temperature dependence of the complex conductivity is compared with those by the BCS model[3][4] and the three-fluid model[5].

### II. MEASUREMENT PRINCIPLE

Figure 1 shows an analytical model of a dielectric-loaded cavity. A cylindrical cavity having diameter  $d$  and height  $h$  consists of a cylinder and an upper plate of perfect conductor with  $Z_s = 0$  and a lower plate of superconductor with  $Z_s$ . A lossless dielectric rod having diameter  $D$ , length

$L$ , and relative permittivity  $\epsilon_r$  is placed in the center of the superconductor plate. For this structure a perturbation formula is given by[6][7]

$$Z_s = R_s + jX_s = \frac{2\pi\mu_0 f_0^2}{\left(-\frac{\partial f}{\partial L}\right)} \left( \frac{1}{2Q_c} - j \frac{\Delta f_0}{f_0} \right) \quad (1)$$

where  $Q_c$  is due to the superconductor plate loss,  $\Delta f_0$  is a frequency difference between the cases of  $Z_s \neq 0$  and  $Z_s = 0$ , and  $(-\partial f/\partial L)$  is the change of  $f_0$  for a small change of  $L$ , which is calculated for three resonator structures; the cavity, the dielectric resonator, and the dielectric-loaded cavity[6][7].

Firstly, when  $\epsilon_r = 1$ ,  $M = 0$ , and  $d = D$  in Fig. 1,  $(-\partial f/\partial L)$  for the cavity is given by

$$\left(-\frac{\partial f}{\partial L}\right) = \frac{c^2}{4L^3 f_0} \quad (2)$$

where  $c$  is the light velocity.

Secondly, when  $d = \infty$  and  $M = 0$  in Fig. 1,  $(-\partial f/\partial L)$  for the dielectric resonator is given by

$$\left(-\frac{\partial f}{\partial L}\right) = \frac{c^2}{4L^3 f_0} \frac{1 + W_d}{\epsilon_r + W_d} \quad (3)$$

Finally, for the dielectric-loaded cavity,  $(-\partial f/\partial L)$  is calculated numerically from the following characteristic equation:

$$\det H(f_0; D, L, M, d, \epsilon_r) = 0 \quad (4)$$

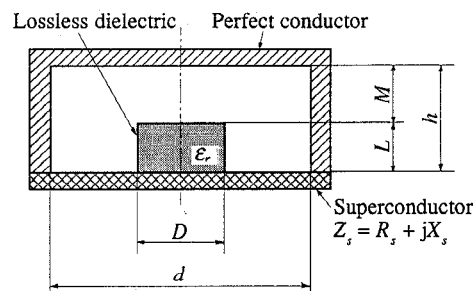


Fig. 1 Analytical model of an TE<sub>01(1+δ)/2</sub> mode dielectric-loaded cavity.

### III. PRECISION OF $Z_s$ MEASUREMENT

For the resolutions of  $Z_s$  measurement we compare three methods described above. The method of the estimation is given elsewhere [7]. Figure 2 shows the results calculated as a function of the dimensional ratio  $S=D/L$  under the following conditions:  $\epsilon_r=24$ , loss-tangent of dielectric  $\tan\delta=4.7\times 10^{-6}$ , and surface resistance of the conductor cylinder and plate  $R_{sm}=7.9\text{ m}\Omega$  at 10GHz. Particularly, for the dielectric-loaded cavity, appropriate values of  $h$  for given  $S$  values were determined so as to minimize the conductor loss. The higher resolving power of  $R_s$  is realized by smaller value of the  $R_s$  resolution in Fig. 2(a). As a result, this method can realize the highest  $R_s$  resolution in these three methods. On the other hand, the higher resolving power of  $X_s$  is realized by greater value of the ratio  $(-\partial f/\partial L)/f_0$  in Fig. 2(b). As a result, this method improves the  $X_s$  resolution by one order of magnitude, compared with the cavity method and is comparable to the dielectric resonator method.

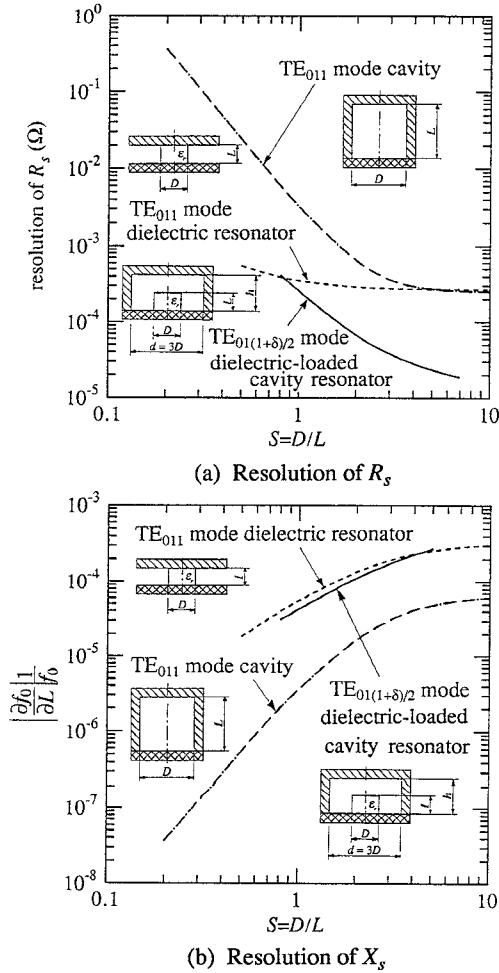


Fig. 2 Resolution of  $R_s$  and  $X_s$  calculated for three resonator methods.  
( $\epsilon_r=24$ ,  $\tan\delta=4.7\times 10^{-6}$ ,  $R_{sm}=7.9\text{ m}\Omega$  at 10GHz)

### IV. MEASURED RESULTS

#### A. Experimental Apparatus

Figure 3 shows an experimental apparatus which is set in the cryostat. This structure is constructed from a copper plated brass cylinder having  $d=18.033\text{ mm}$  and  $h=7.016\text{ mm}$ , two copper plated brass plates, and a BMT ceramic rod having  $\epsilon_r=24.5$ ,  $D=8.007\text{ mm}$ ,  $L=3.328\text{ mm}$ , and the coefficient of thermal linear expansion  $\tau_\alpha=6.3\text{ ppm/K}$  (Murata Mfg. Co. Ltd.). Automatic measurement system of temperature dependence of  $Z_s$  was developed using a HP network analyzer and computer.

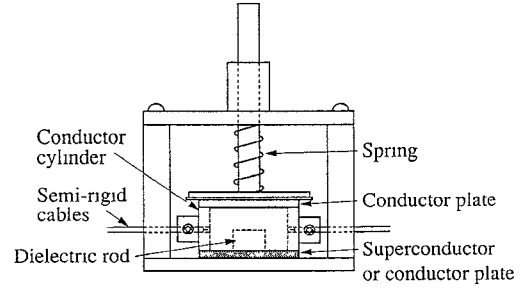


Fig. 3 Experimental apparatus of the dielectric-loaded cavity.

#### B. Temperature Dependence of Parameters for Cavity and Dielectric Rod

In advance, we measured  $d$ ,  $h$ , and relative conductivity  $\sigma_r$  for the copper plated brass and  $\epsilon_r$  and  $\tan\delta$  for the BMT ceramic rod as a function of temperature  $T$ . These results are shown in Figs. 4 and 5. The reproducibility of  $f_0$  for the dielectric-loaded cavity is compared experimentally with one for the dielectric resonator at room temperature. These results are shown in Fig. 6. The reproducibility of measured values for this method is improved over one order, compared with one for the dielectric resonator method.

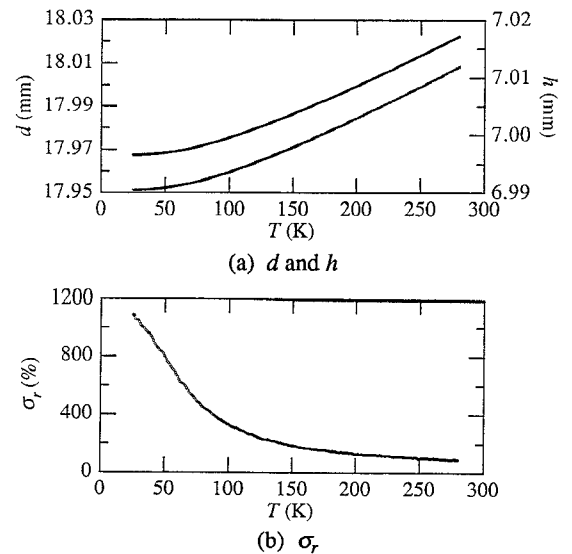


Fig. 4 Measured results of  $d$ ,  $h$ , and  $\sigma_r$  for cavity.

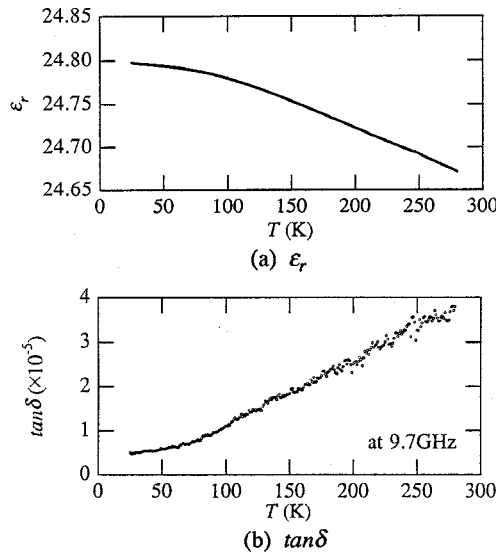


Fig. 5 Measured results of  $\epsilon_r$  and  $\tan\delta$  for dielectric rod.

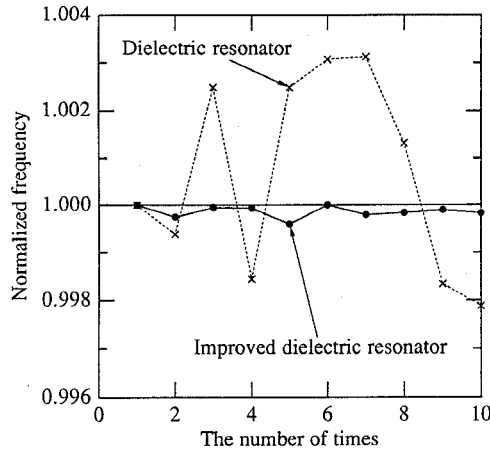


Fig. 6 The  $f_0$  results measured repeatedly by two dielectric resonator methods at room temperature.

### C. Temperature Dependence of $Z_s$ for Superconductors

The temperature dependence of  $Z_s$  were measured for high-Tc superconductors such as Y-Ba-Cu-O (YBCO) bulk, Bi-Pb-Sr-Ca-Cu-O (BPSCCO) bulk, YBCO film of thickness 0.5  $\mu\text{m}$  on MgO substrate, and Eu-Ba-Cu-O (EBCO) film of thickness 0.5  $\mu\text{m}$  on MgO substrate. The measured results of  $f_0$  and  $Q_u$  are shown in Fig. 7. For the film materials, the  $f_0$  and  $Q_u$  values could not be measured in the range  $T > T_c$  because the resonant curve disappeared. For Fig. 7(a), the results and error bars for copper, YBCO, and BPSCCO plates indicate the averages and the meansquare errors of three, three, and two measurements, respectively. Also for three curves, the measured  $f_0$  values decrease with the increase of the scatter. It is expected from this fact that an air gap between the dielectric rod and the conductor plate occurs from the surface roughness of them. The results of surface roughness calculated from the  $f_0$  scatter are compared with

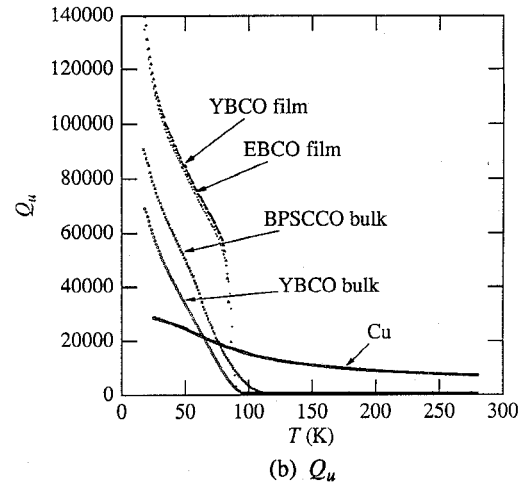
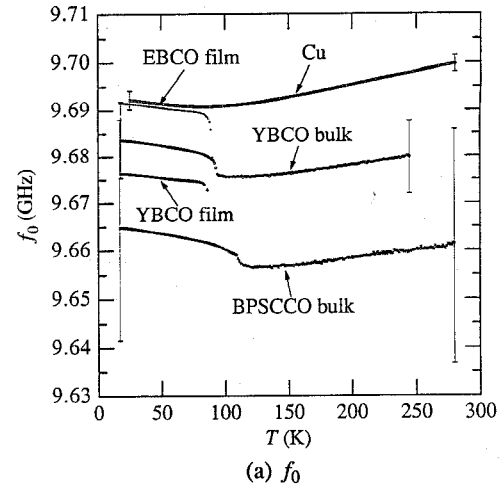


Fig. 7 The  $f_0$  and  $Q_u$  results measured for Cu and some high-Tc superconductors as the lower conductor.

Table 1 Measured results of surface roughness for Cu and superconductors.

Lower plate	Surface roughness calculated from $f_0$ scatter and $f_0$ scatter	Surface roughness measured mechanically*
Cu	1.48 $\mu\text{m}$ (0.00196 GHz)	1.5 $\mu\text{m}$
YBCO bulk	6.25 $\mu\text{m}$ (0.00822 GHz)	4.6 $\mu\text{m}$
BPSCCO bulk	17.6 $\mu\text{m}$ (0.02324 GHz)	13 $\mu\text{m}$

\* included surface roughness of 1  $\mu\text{m}$  for dielectric the ones measured mechanically, shown in Table 1. It is found that the scatter in the  $f_0$  measurements is approximately proportional to the surface roughness measured mechanically. On the other hand, the  $Q_u$  measurements in Fig. 7(b) have the scatter within 1 % and good reproducibility. The  $R_s$  and  $X_s$  values were calculated from Fig. 7 using Eq. (1). These results are shown in Fig. 8. For the films, low  $R_s$  values below  $10^{-3} \Omega$  can be measure at 9.7 GHz, but the  $X_s$  values cannot be obtained because there are not the measured results of  $f_0$  and  $Q_u$  in the range of

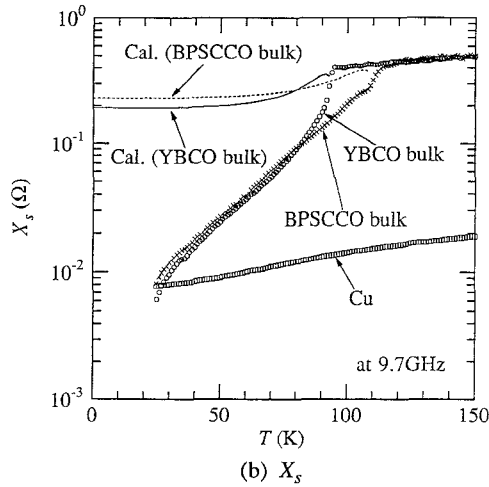
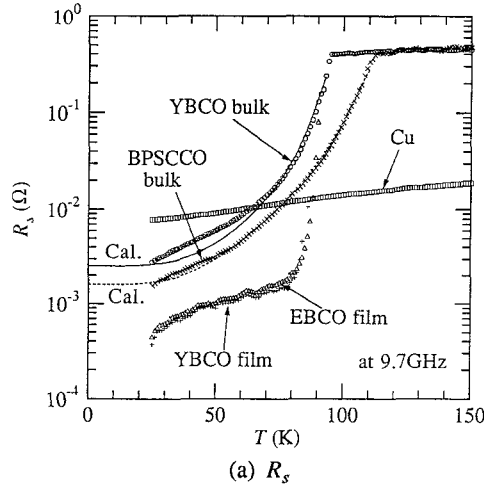


Fig. 8 The  $R_s$  and  $X_s$  results measured for Cu and some high- $T_c$  superconductors and calculated by using three-fluid model.

$T > T_c$ . For the plates, the scatter in the  $R_s$  measurements is within 2 %, and the reproducibility of this method is very good, compared with the scatter of 20 % for the dielectric resonator method. On the other hand, the  $X_s$  measurements in the range  $T < T_c$  have the scatter over one order because of the poor reproducibility in the  $f_0$  measurements. Furthermore, solid and broken curves in Fig. 8 indicate the results calculated by three-fluid model[5][7]. The measured results of  $R_s$  agree well with these calculated ones, while the measured results of  $X_s$  do not agree with the calculated ones because the surface roughness of about  $3\mu\text{m}$  result in difference between the measured and calculated results. Figure 9 shows calculated results of  $\sigma$  for two plates and experimental ones estimated in consideration of the surface roughness. Furthermore, the experimental results of  $\sigma_1$  are compared with ones calculated by BCS and three-fluid models. The experimental results of  $\sigma_1$  near  $T_c$  show the behavior of BCS model as shown in Fig. 10. As a result, three-fluid model is useful in the range of  $T < 0.95T_c$ .

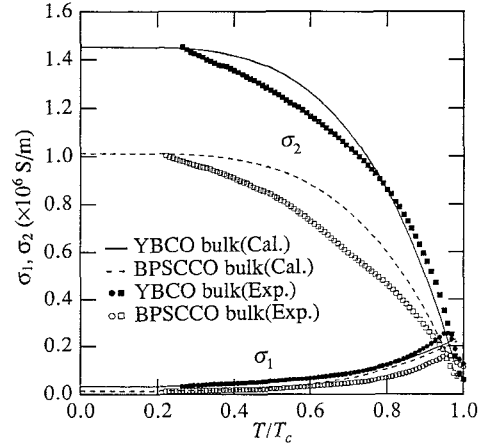


Fig. 9 The  $\sigma_1$  and  $\sigma_2$  results measured and calculated by three-fluid model for high- $T_c$  superconductors at 9.7GHz.

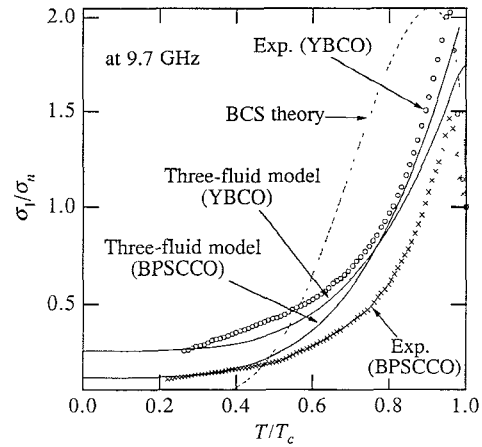


Fig. 10 The  $\sigma_1$  results measured and calculated by BCS and three-fluid models for high- $T_c$  superconductors.

## V. CONCLUSION

The improved dielectric resonator method proposed is useful to measure temperature characteristics of  $Z_s$  automatically with high resolution and good reproducibility. Also, it was verified that surface roughness of materials affects strongly the  $X_s$  measurement.

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